Test of the Kugo-Ojima Confinement Criterion in the Lattice Landau Gauge

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We present the first results of numerical test of the Kugo-Ojima confinement criterion in the lattice Landau gauge. The Kugo-Ojima criterion of colour confinement in the BRS formulation of the continuum gauge theory is given by $u^a_b(0) = -\delta^a_b$, where

$$\int dx e^{ip(x-y)} \langle 0|T D_\mu c^a(x) g(A_\nu \times \bar{c})^b(y)|0\rangle = (g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) u^a_b(p^2). \quad (*)$$

We measured the lattice version of $u^a_b(0)$ in use of $1/(-\partial D(A))$ where $D_\mu(A)$ is a lattice covariant derivative in the new definition of the gauge field as $U = e^A$. We obtained that $u^a_b(0)$ is consistent with $-c\delta^a_b$, $c = 0.7$ in $SU(3)$ quenched simulation data of $\beta = 5.5$, on 8^4 and 12^4. We report the β dependence and finite-size effect of c.

1. INTRODUCTION

The colour confinement problem in the continuum gauge theory was extensively analysed in use of the BRS formulation by Kugo and Ojima[1].

The QCD lagrangian is invariant under the BRS transformation and the physical space is specified as the one that satisfies the condition $\mathcal{V}_{phys} = \{|phys\rangle\}$

$$Q_B|phys\rangle = 0.$$

where

$$Q_B = \int d^3x \Big[B^a D_0 c^a - \partial_0 B^a \cdot c^a + \frac{i}{2} g \partial_0 \bar{c}^a \cdot (c \times c)^a\Big]$$

and $(F \times G)^a = f_{abc} F^b G^c$.

Under the assumption that **BRS singlets have positive metric**, it is proved that $\mathcal{V}_{\mathbf{phys}}$ has positive semidefinite in such a way that **BRS quartet particles appear only in zero norm**.

One finds from the BRS transformation that for each colour a, a set of massless asymptotic fields $\chi^a, \beta^a, \gamma^a, \bar{\gamma}$ form a BRS quartet.

The Noether current corresponding to the conservation of the colour symmetry is $gJ^a_\mu = \partial^\nu F^a_{\mu\nu} + \{Q_B, D_\mu \bar{c}\}$, where its ambiguity by divergence of antisymmetric tensor should be understood, and this ambiguity is utilised so that massless contribution may be eliminated for the charge, Q^a, to be well defined.

Denoting $g(A_\mu \times \bar{c})^a \rightarrow u^a_b \partial_\mu \bar{\gamma}^b$, and then $D_\mu \bar{c}^a \rightarrow (1+u)^a_b \partial_\mu \bar{\gamma}^b$, one obtains the eq.(*) provided A_μ has a vanishing expectation value. The current $\{Q_B, D_\mu \bar{c}\}$ contains the massless component, $(1+u)^a_b \partial_\mu \beta^b(x)$. We can modify the Noether current for colour charge Q^a such that

$$gJ'^a_\mu = gJ_\mu - \partial^\nu F^a_{\mu\nu} = \{Q_B, D_\mu \bar{c}\}.$$

In the case of $\mathbf{1+u=0}$, massless component in gJ'_0 is vanishing and the colour charge

$$Q^a = \int d^3x \{Q_B, g^{-1} D_0 \bar{c}^a(x)\} \quad (1)$$

becomes **well defined**.

The physical state condition $Q_B|phys\rangle = 0$ together with the equation (1) implies that all BRS singlet one particle states $|f\rangle \in \mathcal{V}_{phys}$ are colour singlet states. This statement implies that all coloured particles in $\mathcal{V}_{phys}$ belong to BRS quartet and have zero norm. This is the **colour confinement**.

2. LATTICE CALCULATION OF u^a_b

The Faddeev-Popov operator is

$$\mathcal{M}[U] = -(\partial \cdot D(A)) = -(D(A) \cdot \partial), \tag{2}$$

where the new definition of the gauge field is adopted as $U = e^A$, and the lattice covariant derivative $D_\mu(A) = \partial_\mu + Ad(A_\mu)$ is given in [2].

The inverse, $\mathcal{M}^{-1}[U] = (M_0 - M_1[U])^{-1}$, is calculated perturbatively by using the Green function of the Poisson equation $M_0^{-1} = (-\partial^2)^{-1}$ and $M_1 = \partial_\mu Ad(A_\mu(x))$, as

$$\mathcal{M}^{-1} = M_0^{-1} + \sum_{k=0}^{N_{end}} (M_0^{-1} M_1)^k M_0^{-1}. \tag{3}$$

In use of colour source $|\lambda^a x\rangle$ normalised as $Tr\langle \lambda^a x | \lambda^b x_0 \rangle = \delta^{ab} \delta_{x,x_0}$, the ghost propagator is given by

$$G^{ab}(x,y) = \langle Tr \langle \lambda^a x | (\mathcal{M}[U])^{-1} | \lambda^b y \rangle \rangle \tag{4}$$

where the outmost $\langle\rangle$ specifies average over samples U.

The ghost propagators of $\beta = 5$ and 5.5 are almost the same and they are infrared divergent which can be parameterised as $\dfrac{1}{p^{2.2}}$. We observed that the ghost propagators of $\beta = 6$ is similar to that of $\beta = 5.5$ and its finite-size effect is small[3].

In the similar way, one can calculate the Kugo-Ojima parameter at $p = 0$ as,

$$(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) u^a_b(p^2)|_{p=0}$$
$$= \langle Tr \langle \lambda^a p | D_\mu(A) (\mathcal{M}[U])^{-1} (Ad(A_\nu)) | \lambda^b p \rangle \rangle |_{p=0} \tag{5}$$

We observed that off-diagonal element of u^a_b is consistent to zero, but there are statistical fluctuations. The projection operator $g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$ in equation (*) is treated such that it has an expectation value $\frac{3}{4}$ in the limit of $p_\mu \to 0$.

Making the accuracy of the covariant Laplacian equation solver higher, we observe the tendency that the expectation value of $|u^a_a|$ increases.

At $\beta = 8$, direct measurement of u^a_b gives a large fluctuation, but suitable Z_3 twisting treatment for each sample so that the Polyakov scatter plot should be concentrated around $\arg z = 0$,

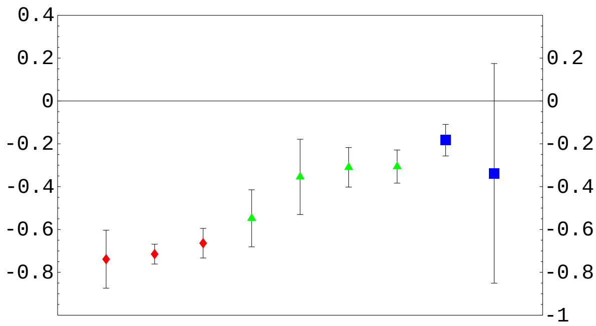

Figure 1. The dependence of space and colour diagonal part of the Kugo-Ojima parameter u^a_a on β and lattice size. (An average over the four directions and eight adjoint representations.) The data points are $\beta = 5.5, 8^3 \times 16; \beta = 5.5, 12^4; \beta = 5.5, 8^4; \beta = 6, 12^4; \beta = 6, 8^4(no\ Z_3); \beta = 6, 8^4(with\ Z_3); \beta = 6, 8^4(with\ Z_3, minimum\ Landau); \beta = 8, 8^4(with\ Z_3); \beta = 8, 8^4(no\ Z_3)$ respectively from left to right.

suppresses the fluctuation and makes the quality of the data better. We consider that this treatment is indispensable in the simulation where Z_3 symmetry persists and the Z_3 factor affects the observed quantity. The similar behaviour is observed in $\beta = 6, 8^4$ lattice. The minimum Landau gauge fixing via smeared gauge fixing performed at $\beta = 6, 8^4$ lattice does not change the expectation value obtained after the $Z3$ twisting but reduces the standard deviation.

The absolute value of u^a_a is plotted as the function of the spatial extent of the lattice aL where a is calculated by assuming $\Lambda_{\overline{MS}} = 100 MeV$. We find for $aL < 2fm$, there exists large finite-size effectDWe expect that by making L large and a small, such that $aL > 2fm$, the absolute value of u^a_a becomes closer to 1.

Non-symmetric lattice $8^3 \times 16$ yields non-symmetric data in μ of (*). This fact shows necessity of tuning lattice constants according to the non-symmetric lattice size and the lattice dynam-

Table 1
Kugo-Ojima parameter u^a_a. Space-diagonal($\mu = \nu$) and off-diagonal components. All data of 8^4 are the average of 100 samples. 'Z3' and 'min' means Z3 twisting and the minimum Landau gauge fixing.

	$diag$	$off-diag$	$diag_1$	$diag_2$	$diag_3$	$diag_4$
$\beta = 5.5, 8^3 \times 16$	-0.739(135)	0.002(60)	-0.776(109)	-0.779(105)	-0.818(118)	-0.581(49)
$\beta = 5.5, 12^4$	-0.715(46)	0.003(32)	-0.729(60)	-0.713(43)	-0.705(39)	-0.712(38)
$\beta = 5.5, 8^4$	-0.664(69)	0.002(45)	-0.669(71)	-0.656(70)	-0.667(67)	-0.664(67)
$\beta = 6.0, 12^4$	-0.548(133)	-0.015(85)	-0.555(123)	-0.561(107)	-0.508(133)	-0.566(159)
$\beta = 6.0, 8^4, with\ Z_3$	-0.303(80)	0.002(29)	-0.286(76)	-0.307(66)	-0.325(81)	-0.293(91)
$\beta = 6.0, 8^4, with\ Z_3, min$	-0.308(88)	-0.000(35)	-0.312(123)	-0.311(78)	-0.317(75)	-0.292(59)
$\beta = 6.0, 8^4, no\ Z_3$	-0.354(176)	-0.001(76)	-0.339(130)	-0.347(161)	-0.378(239)	-0.353(151)
$\beta = 8.0, 8^4, with\ Z_3$	-0.183(74)	0.002(20)	-0.177(71)	-0.197(77)	-0.221(83)	-0.138(19)
$\beta = 8.0, 8^4, no\ Z_3$	-0.338(513)	0.0116(251)	-0.264(278)	-0.359(553)	-0.334(610)	-0.394(536)

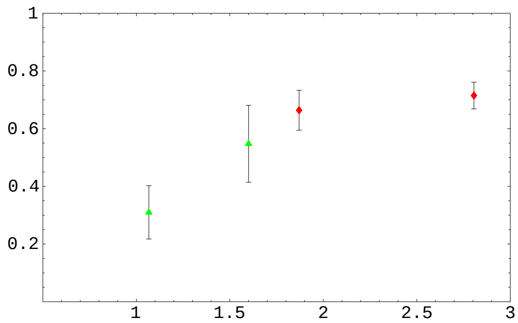

Figure 2. The finite-size effect of the Kugo-Ojima parameter $|u^a_a|$ as the function of the spatial extent of the lattice $aL(fm)$.

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3. SUMMARY AND DISCUSSION

Proof of Kugo-Ojima colour confinement is accomplished successfully only in case of $u^a_b = -\delta^a_b$, and this condition is suggested to be a necessary condition as well. We did the first numerical tests of this criterion by the nonperturbative dynamics of lattice Landau gauge. We observed that the value at $\beta = 5.5$ is around -0.7. Its absolute value decreases as β increases.

We observed the gluon propagator is infrared finite[2] and the ghost propagator is infrared divergent, suggested to be more singular than $\frac{1}{p^2}$, but less singular than $\frac{1}{p^4}$. These results qualitatively agree with the Gribov-Zwanziger's conjecture[4,5], and are consistent with the results of Dyson-Schwinger equation [6]. It is nice to observe that the infrared finiteness of the gluon propagator is in accordance with the Kugo-Ojima colour confinement. As stated in their inverse Higgs mechanism theorem, if we have no massless vector poles in all channels of the gauge field, A^a_μ, and if the colour symmetry is not broken at all, it follows that $1 + u = 0$.[7].

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